**Syllabus***Fall semester, 2018-2019 Academic year*

Academic course information

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| Discipline’s code | Discipline’s title | Type | No. of hours per week | | | Number of credits | | ECTS |
| Lect. | Pract. | Lab. |
| VIMO 4414 | Calculus of variations and optimization methods | MC | 2 | 1 | 0 | 3 | |  |
| Lecturer | S. Serovajsky | | | Office hours | | | Scheduled | |
| e-mail | [serovajskys@mail.ru](mailto:serovajskys@mail.ru) | | |
| Telephone number | +7 701 8315197 | | | Auditory | | | room 4 | |

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| Academic presentation of the course | **Type of university**: The main purpose of the course is to familiarize students with the fundamental concepts of calculus of variations and optimization control theory  **Aim of course:**  as a result of the course students must be able to:  By the end the course, students should be able to:   1. To know the applications of the extremum theory; 2. To know the classification of the problems of the extremum theory; 3. To be able to analyze the extremum of functions; 4. To use the variational method for solving problems of minimization of integral functionals; 5. To know the problem statements of optimal control problems; 6. To know general optimization methods; 7. To know approximation methods for optimal control problems. |
| Prerequisites | Mathematical analysis, functional analysis |
| Post requisites | Differential equations, numerical methods |
| Information resources | **literature**:   1. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011. 2. Serovajsky S. Practical Course of the Optimal Control Theory with Examples. Almaty, Қазақ университеті, 2011. 3. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. 4. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. 5. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001. 6. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 2000. 7. Эльсгольц Л.Э. Дифференциальные уравнения и вариационное исчисление. – М., Наука, 2000.   **Internet-resources:**  [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) . |
| Academic policy of the course in the context of university moral and ethical values | **Academic Behavior Rules:** Obligatory attendance of classes, intolerance for being late, commitment to deadlines for completion and delivery of assignments (CDS, Practical classes, midterm exams, individual projects).  **Academic values:** According to Article 5 of the Code of Honor of students of Al-Farabi Kazakh National University, a student must strictly fulfill his academic duties and prevent academic and legal violations (plagiarism, forgery, use of cribs, deceit of and disrespectful attitude to teaching stuff, absenteeism and coming late without respectful reasons).  All students can receive counseling assistance in person, by phone at the numbers indicated or by e-mail provided. |
| Evaluation and attestation policy | **Criteria-based evaluation:** evaluation of achieving learning outcomes in accordance with the descriptors (checking competencies acquired at weeks of the intermediate control, midterm and final examinations)  **Summative evaluation:**  Final score of the discipline =  IC1, IC2 are intermediate controls, МТ is Midterm, FE – final exam.  Percent-rating letter system for assessing of achievements of leaning outcomes by students:  95% - 100%: А 90% - 94%: А-  85% - 89%: В+ 80% - 84%: В 75% - 79%: В-  70% - 74%: С+ 65% - 69%: С 60% - 64%: С-  55% - 59%: D+ 50% - 54%: D- 0% -49%: F |

Calendar (schedule) the implementation of the course content**:**

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| **Week** | **Topics** | **Hours** | **Max point** |
| 1 | **Lecture 1. Practical examples of the extremum problems**. Maximization of the flight of the body. Brachistochrone problem. Maximization of the flight of the missile. | 2 | 0 |
| **Practical work 1**. Practical examples of the extremum problems. | 1 | 3 |
| **Homework 1**. Practical examples of the extremum problems. |  | 10 |
| 2 | **Lecture 2. Minimization of functions**. Stationary condition. Examples. Maximization of the flight of the body. Minimization of the function of many variables. | 2 | 1 |
| **Practical work 2**. Minimization of functions and stationary condition. | 1 | 3 |
| **Homework 2**. Use stationary condition for the concrete function*.* Check the properties of the stationary points. Chose the function with given property. |  | 10 |
| 3 | **Lecture 3. Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light law. | 2 | 2 |
| **Practical work 3**. Euler equation for Lagrange problem. | 1 | 3 |
| **Homework 3.** Determine Euler equation for the concrete Lagrange problem. Find the general solution of Euler equation, which depends from two constants. Find these constants by means of the given boundary conditions. Find the corresponding solution of the boundary problem. Calculate the corresponding value of the given functional. Calculate the value of the given functional for the linear function which satisfies the given boundary conditions. Compare these results. |  | 5 |
| Independent work of student with teacher-1: Fermat principle and the refraction of light law. |  | 5 |

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| 4 | **Lecture 4. Lagrange problem for the functions family.** Problem statement. The system of Euler equations. Example. Principle of the least action. | 2 | 1 |
| **Practical work 4**. Lagrange problem for the functions family. | 1 | 3 |
| **Homework 4**. Determine the system of Euler equations for the concrete problem. Find general solution of this system. Find the solution of Euler equations, which satisfies boundary conditions. Show the graphs of these solutions. Calculate the corresponding value of the given integral. |  | 10 |
| 5 | **Lecture 5. Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam. | 2 | 2 |
| **Practical work 5**. Lagrange problem with high derivatives. | 1 | 3 |
| **Homework 5**. Determine the system of Euler – Poisson equation for the concrete problem. Find general solution of this equation. Find the solution of Euler – Poisson equation, which satisfies given boundary conditions. Show the graph of this solution. Calculate the corresponding value of the given integral. |  | 5 |
| Independent work of student with teacher-2: Problem statement. Euler – Poisson Equation. |  | 5 |
| 6 | **Lecture 6. Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string. | 2 | 1 |
| **Practical work 6.** Lagrange Problem for functions with many variables. | 1 | 3 |
| **Homework 6.** Determine Ostrogradsky equation for the concrete problem. |  | 10 |
| 7 | **Lecture 7. Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | 2 | 2 |
| **Practical work 7.** Bolza Problem. | 1 | 3 |
| **Homework 7.** Determine Euler equation and the transversality conditions for the concrete problem. Find the general solution of this equation. Find the solution of boundary problem. Calculate the corresponding value of the given integral. |  | 5 |
| Independent work of student with teacher-3: Problem statement. Necessary conditions of extremum. |  | 5 |
| Border control 1 | |  | 100 |
| Midterm | |  | 100 |
| 8 | **Lecture 8. Variational problems with isoperimetric conditions.** Problems with isoperimetric condition. Lagrange multipliers method. A spectrum problem. The problem with many isoperimetric conditions. | 2 | 1 |
| **Practical work 8.** Variational problems with isoperimetric conditions. | 1 | 3 |
| **Homework 8.** Determine Euler equation for the concrete problem. Verify the sign of the Lagrange multiplier with using multiplication of Euler equation by unknown function and integration. Find the general solution of Euler equation; it depends from two constants and Lagrange multiplier. Using given boundary conditions and isoperimetric condition find three unknown constants. Find the set of the solutions of the conditions of the extremum. Calculate the value of the given integral for all solution of the conditions of the extremum. |  | 8 |
| 9 | **Lecture 9. Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum. | 2 | 2 |
| **Practical work 9.** Variational problems with pointwise constraints. | 1 | 3 |
| **Homework 9.** Denote the system of the extremum conditions (concrete Euler equations with boundary and addition conditions). Multiply the first Euler equation by the given parameter a, and second equation by b. Add these equalities with using of the condition (\*). Find Lagrange multiplier λ. Put λ to Euler equations. Find the general solutions of two Euler equations. It equals the sum of the general solution of the corresponding homogeneous equation and the constant, which satisfies the given equation. Find four constants from general solutions of Euler equations with using of the boundary conditions. Put these constants to the formulas of the general solutions. It will be the solution of the problem. | 0 | 3 |
| Independent work of student with teacher-4: Oscillation of the pendulum. |  | 5 |
| 10 | **Lecture 10. Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions. | 2 | 1 |
| **Practical work 10.** Easiest optimization control problems. | 1 | 3 |
|  | **Homework 10.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |  | 8 |
| 11 | **Lecture 11. Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving). | 2 | 2 |
| **Practical work 11.** Optimization control problems for the vector case. | 1 | 3 |
| **Homework 11.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality. |  | 3 |
| Independent work of student with teacher-5: Optimization control problems for the vector case. |  | 5 |
| 12 | **Lecture 12. Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method. | 2 | 1 |
| **Practical work 12.** Optimization control problem with fixed final state. | 1 | 3 |
| **Homework 12.** Determine the function *Н* for the concrete problem. Determine the adjoint system. Determine the maximum principle. Find the control from the maximum principle. Write the iterative method for solving the conditions of the optimality with using of firing method. |  | 8 |
| 13 | **Lecture 13. Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals. | 2 | 2 |
| **Practical work 13.** Differentiation of functionals and abstract optimization problems. | 1 | 3 |
| **Homework 13.** Calculate Gateau derivative for the concrete functional. Determine gradient method and projection gradient method. |  | 3 |
| Independent work of student with teacher-6: Differentiation of functionals and abstract optimization problems. |  | 5 |
| 14 | **Lecture 14**. **Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional. | 2 | 1 |
| **Practical work 14.** Variational inequalities. | 1 | 3 |
| **Homework 14.** Calculate Gateau derivative for the concrete functional. Determine variational inequality. Find the solution of variational inequality. Calculate the value of minimizing functional. |  | 8 |
| 15 | **Lecture 15. Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example. | 2 | 2 |
| **Practical work 15.** Existence and uniqueness of extremum problems. | 1 | 3 |
| **Homework 15.** Prove the convexity and continuity of the concrete functional. Prove the convexity, closeness and boundedness of the concrete set of admissible control. Prove the existence of the optimal control. |  | 3 |
| Independent work of student with teacher-7: Existence theorem for abstract optimization problems. |  | 5 |
| Border control 2 | |  | 100 |
| Total | |  | 100 |

Lecturer S. Serovajsky

Head of the department of Kh. Khompysh

Chairman of the Faculty Methodical Bureau U.R.Kusherbayeva